arq.

Ponto IV
1963

AGENCY FOR INTERNATIONAL DEVELOPMENT
UNITED STATES A.I.D. MISSION TO BRAZIL
c/o American Embassy

Rio de Janeiro, Brazil

22 de maio de 1963

Exma. Sra.
D. Lúcia Marques Pinheiro,
MD Diretora da DAM
INEP
Ministério da Educação e Cultura
Rio de Janeiro

M. E. C.
INSTITUTO NACIONAL DE ESTUDOS PEDAGÓGICOS
2
PROTOCOLO
1250/63

Alice
24/5/63

Prezada D. Lúcia:

Em atendimento ao pedido constante de sua carta nº 248, de 26 de março de 1963, estamos anexando à presente, para sua informação, cópias das cartas que recebemos em resposta às que enviamos solicitando material sôbre o Ensino da Matemática na Escola Primária.

Seguem, também, em anexo, as publicações recebidas de algumas das instituições com que nos comunicamos.

Estamos aguardando resposta de duas universidades e, tão logo recebamos qualquer comunicação, entraremos novamente em contacto com V. Sa.

Atenciosamente,

Joseph E. Albers

Joseph E. Albers
Program Assistant

mgc
Anexos

STATE OF MINNESOTA
DEPARTMENT OF EDUCATION
CENTENNIAL OFFICE BUILDING
ST. PAUL 1, MINNESOTA

April 10, 1963

Mr. Joseph E. Albers
USAID/ED
APO 676
New York, New York

Dear Mr. Albers:

This is an answer to your letter of April 5. We have no guides in the Minnesota State Department of Education for teaching of arithmetic. I think that many of the textbook companies would be able to provide this type of service.

Last month, Miss Norma Osorio of Brazil visited with me, and at that time she picked up quite a bit of information and pamphlets which were of interest to her. Extra copies of these may be obtained by writing to the Minnesota National Laboratory.

Sincerely,

David Dye
Mathematics Consultant

DD:rc

URBANA, ILLINOIS DAVID A. PAGE, DIRECTOR

1207 W. Stoughton
April 15, 1963

Mr. Joseph E. Albers
Program Assistant
USAID/ED
APO 676
New York, New York

Dear Mr. Albers:

Under separate cover, I am sending you complimentary copies of materials currently available from this Project. I hope they will be of value to you. Due to a surge of orders for copies of Number Lines, Functions, and Fundamental Topics, our supply is exhausted. A revision of this book is now in production by the Macmillan Company and it should be available within a few months. You will be notified by the Macmillan Company when the book is ready for distribution.

Your name is now included on our mailing list. You will receive announcements of further publications and activities of the Project. Let me know if I may be of further help to you.

Sincerely yours,

David A. Page

David A. Page

DAP:th

Sent separately: Information Sheet (3) ✓
Estimation (3) ✓
Arithmetic with Frames, rev. (3) ✓
Hohn Reprint (3) ✓
NL Review (3)
NL Table of Contents (3) ✓
NL for the Orbiting Atomic Teacher (3) ✓
Well-Adjusted Trapezoids (3) ✓
A First-Grade Sample (3) ✓
Maneuvers on Lattices (3) ✓

Received 5/22/63

SCHOOL MATHEMATICS STUDY GROUP
SCHOOL OF EDUCATION
STANFORD UNIVERSITY
STANFORD, CALIFORNIA

E. G. BEGLE
Director

May 8, 1963

Mr. Joseph E. Albers
Program Assistant
USAID/ED
APO 676
New York, N.Y.

Dear Mr. Albers:

Dr. Begle has asked me to reply to your letter of April 3.

The School Mathematics Study Group has published sample textbooks for grades 4 through 12 reflecting the most modern concepts of mathematics education. Those for grades K - 3 are now in preparation. These books, with teachers's commentaries may be purchased according to the instructions in the enclosed newsletter.

You may be interested in knowing that the texts for grades 7 through 10 have already been translated into Spanish and those for other grades are in preparation.

Yours sincerely,

Max Kramer

Max Kramer
Project Coordinator

MK:sg

Enclosure

MADISON PROJECT

SYRACUSE UNIVERSITY
WEBSTER COLLEGE

April 9, 19[illegible]

Mr. Joseph E. Albers
[illegible]
APO [illegible]
New York, New York

Dear Mr. Albers:

As Dr. Davis is rarely in Syracuse this year, I am taking the liberty of answering your letter of April 3, 1963.

We are forwarding to you, under separate cover, complimentary ~~copies~~ of of the three Madison Project publications, Discovery in Algebra, Axioms for Arithmetic and Algebra, and Matrices, Functions, and Other Topics. I hope you will find these of interest and use in your work.

I am also enclosing a price list of presently available materials. Should you be interested in borrowing any of the tape recordings, or securing further information on the films, please feel free to contact us.

Cordially,

(Mrs.) Marilyn Hurley
Administrative Assistant

Received 5/22/63

STANFORD UNIVERSITY
STANFORD, CALIFORNIA

INSTITUTE FOR MATHEMATICAL STUDIES
IN THE SOCIAL SCIENCES
Ventura Hall

April 9, 1963

Mr. Joseph E. Albers
Program Assistant
USAID/ED
APO 676
New York, New York

Dear Mr. Albers:

In reply to your letter of April 3, enclosed is a description of the Sets and Numbers project here as well as a project in mathematical logic, together with a sheet giving instructions for ordering material. I am sorry not to be able to send sample textbooks gratis, but the demand on project funds far exceeds our ability to distribute material without charging for it.

Sincerely yours,

PS/lt

Patrick Suppes

Suppes Arithmetic Project
Ventura Hall
Stanford University
Stanford, California

Sets and Numbers, Books 1A and 1B, constitute a complete curriculum for the first grade; Sets and Numbers, Books 2A and 2B, constitute a complete curriculum for the second grade. L. W. Singer Company, 249 West Erie Boulevard, Syracuse 2, New York, are publishing the Sets and Numbers books for the first and second grades, as well as Sets and Numbers, Kl, for kindergarten children. Please contact them directly for the kindergarten, first and second grade Sets and Numbers books and their accompanying teacher's manuals.

The supply of the preliminary edition of Sets and Numbers, Book 3A, the first half of the third grade, is now exhausted. However, the preliminary edition of Sets and Numbers, Book 3B, the second half of the third grade, is now available through Suppes Arithmetic Project.

Mathematical Logic for the Schools, Book I, is designed to provide a complete course for selected and talented students in any of the Grades 5 to 9, and for regular high school students. It is now available from Suppes Arithmetic Project. A manual of solutions to the old edition is available.

A half-hour film (16 mm. sound) giving a demonstration with four of the fifth-grade students in the Mathematical Logic program is also available at a rental cost of $10 plus postage.

If you would like to purchase Sets and Numbers, Book 3B or Mathematical Logic for the Schools, please send the correct amount to me at the above address. A reduction in price will be made for orders of twenty-five copies or more of Book 3B. Please make check payable to Suppes Arithmetic Project. The price for individual copies is as follows:

Sets and Numbers, Book 3B	$1. 50
Teachers's Manual for 3B (available approximately April 1963)	. 50
Mathematical Logic for the Schools	2. 50
A Solution Book to the old edition	. 50

Copies of the first two books of the geometry series, Geometry for Primary Grades, Books I and II, written by Newton Hawley and Patrick Suppes, may be obtained directly from Holden-Day, Inc., 723 Montgomery Street, San Francisco, California.

Louise Thursby

(Mrs.) Louise Thursby
Administrative Asst.

California residents add four percent sales tax.

MATHEMATICAL LOGIC FOR THE SCHOOLS

An Experimental Mathematics Project

for the Elementary School

1961-62

Patrick Suppes

Project Director

December 1, 1962

Institute for Mathematical Studies in the Social Sciences

Applied Mathematics and Statistics Laboratories

Stanford University

Stanford, California

1. Statement of Project Goals.

In general terms the aim of the program is to deepen and extend the mathematical experiences of the able elementary school child at the broadest level of mathematics, the level of methodology and the theory of proof. The approach is through a study of modern mathematical logic, in particular that portion of it which is concerned with the theory of logical inference or the theory of deduction. In addition the theory of logical consistency is also studied.

In modern times logic has become a deep and broad subject. Only in recent years have the systematic relations between logic and mathematics been established and a completely explicit theory of inference formulated which was adequate to deal with all the standard examples of deductive reasoning in mathematics and the empirical sciences.

The concept of axioms and the deduction of theorems from axioms is at the heart of all modern mathematics. The purpose of this project is to introduce the academically gifted elementary school child to modern mathematics and mathematical methods at a level which is rigorous but simple enough in presentation and context to permit comprehension by elementary and secondary school students.

Specifically, the objective of the project is to experiment on the feasibility of teaching mathematical logic to classes of academically talented fifth and sixth grade children. Thus it is considered a program in enrichment for the more able mathematics student and a supplement to the elementary school mathematics curriculum. Questions of particular interest involve the capacity of children of the fifth and sixth grade age level to do the kinds of deductive proof which are characteristic of modern mathematics, specific factors of difficulty, and the possible transfer of skills of analysis and correct reasoning to other subject matter areas.

The project also seeks to learn how much specific teacher training is needed.

In the study of logic, the child is introduced to a way of using language and ideas precisely. The emphasis is upon clarity and precision in thinking, upon rigor and consistency. The student is learning to recognize the logical structure of English sentences and of discourse. The introduction of the concept of form and structure in language, and analysis of structure are integral parts of the child's study. Thus it is clear that while this project is considered a supplement to the program in mathematics in the elementary school, the application of the concepts and skills learned is not limited to this subject. The principles of logical inference are universally applied in every branch of systematic knowledge.

Logic has always been considered a college-level subject. The belief has been that it is too abstract for the understanding of the elementary school student. Evidence is mounting that logic is not too abstract for the elementary school child of age 10, 11 or 12 but on the contrary, that this age may represent a most propitious time for the introduction of abstract concepts.

The importance of the theory of proof and of the methodology of deriving theorems from axioms in modern mathematics cannot be questioned. Yet development of the skills of deductive reasoning has been left largely to incidental learning in the school mathematics curriculum. The point of view represented in this program is that deliberate and well-planned teaching of mathematical logic will enlarge the scope of the able student both in mathematics and other fields of systematic knowledge and will provide a background for a deeper and more penetrating study of mathematics and other systematic studies later.

2. Pilot Study: 1960-61.

A pilot study in teaching mathematical logic to a group of selected fifth-grade students from a public school near Stanford University was carried on during the academic year, 1960-61. The class consisted of twenty-five students who were selected from the several fifth-grade classes in the school on the basis of their arithmetic achievement scores. The children were given the arithmetic achievement test during the seventh month of the fourth-grade year. All students selected for the logic programs scored at the sixth-grade level or above. The class comprised approximately 27 per cent of the total fifth grade in the school.

The class met three or four days per week throughout the school year for periods of about thirty or thirty-five minutes each. It was taught by Dr. Shirley Hill, a member of the Stanford Project staff. Textbook material, composed of explanatory material and practice exercises, was written for and used by the class. Experimentation with this material and subsequent revisions resulted in a textbook in mathematical logic for the schools. This book, revised in 1962, will be discussed in detail later in this report. The content covered by the pilot class is that which is contained in the first six chapters of the textbook.

Achievement tests given periodically indicated that this material is well within the capacity of the able fifth-grade student. Although the class proceeded much more slowly than the usual undergraduate class in college mathematical logic, the results indicated that children of this age are able to achieve a level of accomplishment comparable to college students in the skills of deduction essential to mathematical reasoning when instruction is geared to their level of experience.

3. Experimental Program: 1961-62.

Eleven experimental classes of selected fifth-grade students began the work in mathematical logic in September, 1961. These classes represented eleven different schools in six school districts in the San Francisco Bay Area. In each school approximately 25 to 30 per cent of the fifth graders were selected by the administrators on the basis of general ability and high achievement in mathematics.

Each of the classes met three days a week for a period of between 30 and 40 minutes. The groups were taught in each case by a classroom teacher from the staff of the school. During the summer of 1961, these eleven teachers completed a four-weeks intensive course in logic at Stanford University. The course was an introduction to mathematical logic with special emphasis on problems of teaching the subject matter to elementary school students. The book, Mathematical Logic for the Schools, was used in the summer course and was distributed to all experimental classes.

The organization of the work in the experimental classes followed the sequence presented in the textbook. There was, however, no effort to maintain a uniform rate of progress in all classes. The teachers were not committed to covering any specific body of content during the first year, but were encouraged to proceed at the pace they felt to be most appropriate for their classes. During the school year, 1961-62, the original pilot class continued in the sixth grade portion of the program as a pilot group taught by Frederick Binford of the project staff. Additional materials were developed for this class and this experimentation will lead to the second book in the series, Mathematical Logic for the Schools, now in preparation.

4. Experimental Program: 1962-1963.

For the academic year, 1962-63, the program in mathematical logic is continuing with the same experimental group now at the sixth-grade level. In addition, a new group of fifth-grade experimental classes began the program. The plan is to continue to train teachers for the experimental classes.

5. Testing Program.

In order to evaluate the achievement level of the experimental classes, a series of tests was given to the experimental group and results compared with those from the same test given to the regular undergraduate college classes in mathematical logic at Stanford University during the spring quarter, 1962. The college group used the same text book, Mathematical Logic for the Schools. These tests were examinations of achievement in the subject matter of mathematical logic. Evidence indicates that the level of accomplishment of the elementary school students is comparable to that of a college class in mathematical logic although their pace is slower. However, it should be pointed out that the college students put in about ten times as many hours each week on logic as do the elementary school students. So, although the college students achieve about as much in four weeks as do the elementary school students in an academic year, their achievements per hour are quite comparable. It is suggested that the two groups, experimental and control, may be considered comparable in that both represent a selected group in terms of the general population. It is understood that this comparison of achievement is not at comparable time periods but at comparable points of accomplishment in covering the systematically organized body of course content.

Three tests were given to the college group of 186 students. The first of these tests was also given to all eleven fifth-grade classes, 260 students. The test was on the recognition of the logical structure of sentences and early work in symbolic deduction. A few very low scores pulled the mean well below the median. For the college group: mean 91, median 97. For the fifth grades: mean 76, median 83.

Seven fifth-grade classes (164 students) took the second test. This tested mostly the symbolic manipulations involved in deductions. The results were for the college group: mean 70, median 72; for the fifth-grade group: mean 67, median 77. The mean scores of the fifth-grade classes were: 92, 87, 79, 78, 76, 49, and 36--five of the seven above the college mean.

TABLE I

TEST 1 POSSIBLE SCORE - 100

	N	MEAN	MEDIAN	RANGE
College Students	186	91	97	8-100
Fifth-Grade Students	260	76	83	17-100

TABLE II

TEST 2 POSSIBLE SCORE - 100

	N	MEAN	MEDIAN	RANGE
College Students	189	70	72	19- 99
Fifth-Grade Students	164	67	77	6-100

These test results suggest that the symbolic manipulations characteristic of formal proofs are relatively easier for elementary school children than the problem of recognizing the logical form or structure of sentences in ordinary English. Both the basis of this conclusion and its implications for the organization of mathematics instruction are being further investigated.

The testing program is continuing in 1962-1963. Another control group of college students has been tested. As the experimental classes of students now in sixth grade advance, they will take the same tests. At the same time beginning classes of fifth grades are also joining the program and will be tested.

In addition to the achievement tests in the subject matter itself, general achievement tests will be given at the end of the second year. The purpose of the general achievement tests is to test for transfer and generalization, and these will be given to the experimental group and a matched control group of the same age and general intelligence from the same or similar school districts.

6. Teacher Training.

The teachers of the experimental classes received a first course in mathematical logic at Stanford University during the summer of 1961 or the spring of 1962. In addition, they are taking a second course during the autumn of 1962, concurrently with teaching their first- or second-year group of elementary school students.

It is intended that close contact be maintained between members of the project staff and the teachers of the experimental classes. During the 1961-1962 academic year, the teachers and project staff met in monthly meetings. During the autumn of 1962, these meetings have been suspended since the teachers meet with the staff in class every week. In addition, members of the staff continue to visit every class several times and to consult with the teachers.

7. The Textbook.

The textbook used by all classes is Mathematical Logic for the Schools, by Patrick Suppes and Shirley A. Hill. It contains explanatory material with an emphasis on exercises and examples. The organization of the book and its content is outlined below.

The book is self-contained but has more material than was covered by any of the classes in 1961-1962. The sixth-grade classes in the autumn of 1962 have spent varying lengths of time reviewing and have then simply continued in the book.

During the summer of 1962, a revision of the textbook was prepared on the basis of experience with its use in 1961-1962. Since the book had proved satisfactory for the college control group as well as the fifth-grade students, the revision was prepared with the intention of making it suitable for students in secondary as well as elementary school.

TABLE OF CONTENTS

In addition to the text a solutions manual has been prepared by Frederick Binford containing complete solutions to all of the exercises in the text, some further discussion and suggestions for the teacher and explanations of the more difficult solutions. The title of this work is Manual of Solutions to the Exercises in Mathematical Logic for the Schools, Book I. Both it and the text are being published in 1963, by Blaisdell Publishing Company, 501 Madison Avenue, New York 22, New York. The preliminary edition of the book (but not the Manual of Solutions) may be purchased at cost from the Suppes Arithmetic Project, Ventura Hall, Stanford, California.

The second volume of Mathematical Logic for the Schools by Patrick Suppes and Frederick Binford is now in preparation and will be available in preliminary edition; in the summer of 1963.

8. Staff.

The director of the project is Dr. Patrick Suppes, Professor, Philosophy and Statistics, Stanford University. His research associates are Dr. Shirley A. Hill and Mr. Frederick Binford of the Institute for Mathematical Studies in the Social Sciences and Laboratory for Quantitative Research in Education, Stanford University. Inquiries about the program should be addressed to Professor Suppes, Ventura Hall, Stanford University, Stanford, California.

9. Project Support.

The project is supported by grants from the United States Office of Education and from the National Science Foundation. The textbooks are distributed free of charge to the participating classes.

SETS AND NUMBERS

An Experimental Project in the Teaching of
Elementary School Mathematics
1962-1963

Patrick Suppes
Project Director

January 1, 1963

Institute for Mathematical Studies in the Social Sciences
Stanford University
Stanford, California

SETS AND NUMBERS

An Experimental Project in the Teaching of Elementary School Mathematics

1. Objectives. The primary objective of the project is to develop and test materials for a mathematics curriculum for grades K through 6. The intention is to provide a program which is both mathematically sound and pedagogically simple.

The curriculum is expanded and extended at every grade level. Significantly more content is included than has been included in the traditional program. Although the major emphasis is on the development of the concepts, laws, and skills of arithmetic, considerable content from other branches of mathematics is added. For example, the inclusion of a substantial body of content from geometry is introduced by work with simple geometric constructions in the first, second, and third grades. As another example, by introducing the use of letters as variables, the present program includes the basis for a smooth transition to the study of algebra, a step which has been difficult for many children to make. In presenting letters as variables in a simple context that requires no technique of solution, a familiarity with algebraic variables may be developed at the earliest stages of the child's mathematical education.

While the project takes the point-of-view that there are many sound and valid arguments for the addition of more and different content in elementary school mathematics, and that evidence is ample that children of this age can learn much more mathematics than traditionally assumed, a goal of greater importance is that of providing a stronger foundation in arithmetic. Arithmetic is taught with an emphasis on concepts, on structure and logical development, on laws, without sacrificing the development of skills. The goal is to deepen as well as to extend mathematical experience of the child.

The materials stress precise and exact mathematical language. Vague and ambiguous terms are avoided and technical vocabulary is used where appropriate. Experience has shown that technical vocabulary is easily learned by the young child when the idea represented is clear and explicit.

Particular attention is paid in the material to the sequence of development of concepts. The attempt is to move from the concrete to the abstract, from familiar ideas to new ideas in a series of small steps, each one building on the previous one.

2. <u>The concept of a set and its role in the program</u>. The central concept in the project materials developed for primary grades is that of a set. It is the foundational and unifying idea throughout. This concept is basic to the development of the idea of a number and operations on sets are introduced as fundamental to the parallel operations on number.

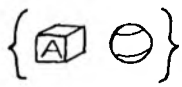
All mathematics can be developed from the concepts of set and operations upon sets. The study of sets would therefore seem to provide the ideal initial introduction to mathematics but is ordinarily considered too "abstract" to introduce early in the student's development. The term "set" is in fact a simple concrete term--easy both to explain and understand. A set is simply any collection or family of objects. Thus, we may speak of the set or collection of all students now in the first grade, the set or collection of dolls now owned by Mary Jones, the set or collection of all Irishmen, the set or collection of all cowboys. The method of displaying problems by grouping objects as used in the standard workbooks approaches the set notion, but the lack of an explicit notation makes precise definitions difficult, if not impossible. Our empirical findings indicate that simple mathematical concepts can be understood by the beginner provided that these are presented precisely with the help of a consistent notation.

There are at least two major reasons to begin first-grade arithmetic with the explicit introduction of the notion of set and appropriate notation for sets and operations upon them. In the first place, sets are more concrete objects than numbers. At the same time operations upon sets are more meaningful to the student than manipulation of numbers. The putting together of sets of physical objects, for example, is a more concrete operation than the addition of numbers. The many exercises in the grouping of objects displayed in current workbooks is in fact a recognition of the greater concreteness.

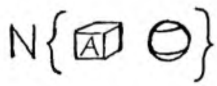
In the second place, the prior introduction of sets and additional explicit notation permits mathematically exact and precise definitions and concepts rather than the often vague and ambiguous notions encountered in explanations of relations between concrete groups of objects and the Arabic numerals. For example, students can learn a clear, simple, and meaningful characterization of numbers as properties of sets. Children who have learned the notion of a property can learn that a precise answer to the question "What is a number?" is, "A number is a property of a set." Continual emphasis on this concept of numbers precludes learning number symbols as meaningless symbols or addition as a meaningless operation performed on meaningless number symbols. Numbers are properties of sets and the operation of addition of numbers is simply a general way of combining families of sets of things without paying any real attention to the things themselves.

With this introduction of number, zero presents no special difficulty and takes on no special mystique but is introduced as the number property of the empty set, the set which has no members. Empirical evidence strongly suggests that less difficulty is encountered with zero in arithmetical operations than any number when it is introduced as a property of the empty set.

The introduction of explicit notation makes the concept of number clear. Ordinarily, we can only assume understanding of the relationship when we make the great leap in abstraction from groups of objects to numerals which name their particular number properties. The use of set notation allows the steps in abstraction to be made explicit. The first step in abstraction is describing a set in the following way:



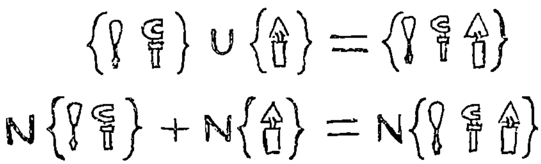
The next step in abstraction is the N notation:



This notation names a number but at the same time it maintains the pictorial character of the set description. In this sense it may be considered a "transition" to the Arabic numerals. Here we have abstracted from the particularity of the objects in the set to the single property of number. We must assume that children understand this step if they are to have any understanding of the way a number is related to a set of objects. The notation makes the step clear and precise in a way not permitted by verbalization at the primary grade level. The final step is to the Arabic numeral which in our example is:

2

Thus the explicit notation for sets introduces the student at the very beginning of his mathematical experience to the easily comprehended operations on sets--rather than to the more difficult and more abstract operations on numbers. Moreover the introduction of notation for sets permits consideration of addition and subtraction of numbers without commitment to the particular notation of Arabic numerals. The possibility of breaking free from the reiterated use of this single notation for numbers does much to eliminate the tendency to focus on the numerals themselves without consideration of their meaning. To illustrate these points about concreteness and meaningfulness the introduction of addition at three levels is illustrated below.



$$2 + 1 = 3$$

In the first line the concrete operation of putting sets together, that is, of forming their union, is indicated. In the second line the notation of N in front of a set is used to indicate the number of objects in the set. This notation permits a very simple transition from the concrete putting together of objects to the abstract use of Arabic numerals, introduced in the third line.

Throughout the beginning materials, set operations are presented as concrete analogues to numerical operations. Union of sets and addition of numbers are presented in sequence, difference of sets and subtraction of numbers are presented in sequence, etc.

Compared to most books, the present program introduces an extensive mathematical vocabulary and notation. It is believed that the use of explicit notation permits a clarity of definition of concepts that is otherwise impossible so that the student can develop a deeper understanding of number concepts. The essential nature of notation for precision in mathematics cannot be overlooked for it is a truism of the history of mathematics that many strides of major importance have depended almost solely on the introduction of an appropriate mathematical notation. It is indeed difficult to penetrate far in mathematics without the development of an explicit notation.

Experimental work with the material during the academic years, 1959-60, 1960-61, and 1961-62 has shown that primary-grade children do not have difficulty in grasping new notation. As long as the notation introduced is explicit and precise and corresponds to simple notions, no difficulties of comprehension arise. The notation and vocabulary introduced in the program have already been well tested and have been found to be well within the capacity of the primary-grade child.

It should be emphasized that it is not our intention to present an isolated body of mathematical content, but rather to present basic concepts and mathematical tools with which an integrated program of learning mathematical tools can be constructed.

Finally, it is believed that the introduction of such basic concepts as those of set and set operations will lead to greater understanding of the structure of the mathematics to be learned.

3. Experimental Program - 1960-1961. During the academic year, 1960-1961, the program in Sets and Numbers was carried on in twenty-five first-grade classes from five school districts in the San Francisco Bay Area. It comprised the total arithmetic program for each of the classes during the entire school year. The twenty-five classes represented a full range of ability levels. Eight classes were in schools which group classes homogeneously in terms of general ability and all first graders within those schools participated in the experiment. The remaining seventeen groups were hetereogeneous in ability.

The workbooks, Sets and Numbers, Book 1A and Sets and Numbers, Book 1B were used by all experimental classes. In addition, classes which completed these two books worked in a book of optional material. Although the sequence of the concepts introduced remained constant, the rate at which the classes proceeded varied considerably. The determination of the pace at which a class would work was the responsibility of the classroom teacher. There was no effort to maintain uniformity in the amount of time allotted to any one section of the workbooks.

Classes were taught by the regular classroom teachers. No special training or background was presupposed. The teachers met for a general orientation at the beginning of the school year and monthly thereafter for discussion of progress, introduction of new materials, etc.

At the end of Book 1A, 17 experimental classes were tested on the materials covered in that book to determine whether children of first-grade age could learn the concepts and skills introduced and to obtain some evidence as to the relative difficulty of these concepts and operations.

The test consisted of 73 items covering all the types of problems which appeared in Book 1A. Tests were administered by the classroom teachers. With a possible score of 73, the mean score was 61.40. The median was 65. An analysis of errors on each item indicated the relative difficulty of types of items. Specific analyses of data will not be reported here, but in general, it is clear that the notion of set and the operation on sets are comprehensible to first graders and are somewhat less difficult than operations on numbers.

Individual scores were broken down by ability level where classes were homogeneous into a lower one-fourth, a middle one-half, and an upper one-fourth categorization. It is interesting to note that although on all items the low group made more errors than the high group, the difficulty of an item relative to other items was approximately the same for all groups. The proportion of total errors made on any one type of item varies little over the three ability levels, that is, the order of difficulty is approximately the same.

A few of the results which might be noted are: problems which are most difficult are two-step problems such as '2 + ___ = 3 + 1', and problems which mix two types of notation; use of variables did not increase the difficulty of work with equations; the least difficult items were invariably those using the empty set or the number 0 .

Even though many of the concepts introduced in the program are new to the first-grade curriculum, the project is also committed to the objective that the experimental classes will achieve at least the same level of proficiency as non-experimental classes on other concepts ordinarily taught in more conventional programs. But it is obvious that the rationale behind the experimental program suggest the prediction that a higher level of proficiency in work with numbers will be achieved than is achieved by classes with more traditional approaches.

At the end of the school year, a test of general achievement was given to all experimental classes and to an equal number of comparable non-experimental classes within the same districts. The classes (which will be referred to here as control classes) were matched with the experimental classes by the districts' administrators. The administrators' choices were based upon their judgment of similar levels and range of ability, of comparable socio-economic background, and any other relevant factors of which they were aware.

The test instrument covered the concepts found within most first-grade arithmetic books, with no material which is unique within the Sets and Numbers program. It included some problems based on concepts covered in any first-grade curriculum but in forms which were unfamiliar to both experimental and control groups. Thus some items required the children to apply what they had presumably learned to problem forms which were new to them.

A full report of results is not included here, but a comparison of mean scores and medians is reported in Table 1 below. The total possible score on the test was 118.

Table 1

Comparative Scores on General Achievement Test, May 1961

	N Students	N of Classes	Mean	Median
Experimental Group	595	25	97.39	104
Control Group	539	25	86.68	90

The difference between means, in favor of the experimental group, was significant, with $P < .01$.

In terms of the proportion of children scoring correctly on individual items, the results can be broken down in the following ways:

(i) There were not significant differences between experimental and control groups on items involving

a. simple recognition of groups and of Arabic numerals

b. sequence of numerals

c. telling time, ordinals, and fractional part (half).

(ii) The experimental group was superior on items involving

a. decomposition of tens and ones, place value

b. writing numerals

and markedly superior on

c. items involving arithmetical operations (addition and subtraction, in column or in equation format, in several degrees of complexity and in several forms).

It should be mentioned that the enthusiastic response of teachers and administrators in assessing the results of the first years' experimentation was a considerable part of the preliminary evaluation. The majority of classroom teachers involved were very favorable in their own evaluations of the workbooks. All participating school districts chose to continue with the program and most of the districts requested that additional schools be included. It may also be of interest to note that all teachers reported a high level of motivation and interest among the children at all levels of ability.

In addition to the classroom experimentation, a supplementary experimental program concerned with the detailed analysis of the formation and learning of mathematical concepts in young children is underway. In this part of the program, intensive study is made of individual children in highly controlled experimental conditions in order to analyze the acquisition of the concepts introduced in the workbook materials. Particular attention is given to sources of greatest difficulty. Experiments of this character will contribute to the development of future materials and have contributed to changes in the existing books.

4. Experimental Program - 1961-1962. For the academic year, 1961-62, the experimental classroom program was considerably expanded. The original twenty-five first-grade classes continued and used the second-grade workbooks. In addition, twenty new second-grade classes which had not worked previously in the *Sets and Numbers* materials began the program with second-grade workbooks. *Sets and Numbers, Book 2A* contains an extensive review section for such classes.

At the first-grade level, eighty classes began work on the revised first grade materials. Thus, 80 first-grade and 45 second-grade classes participated in the classroom testing of materials in 1961-62 under the direct supervision of the project staff. This group included the original five districts plus two additional San Francisco Bay Area districts, and districts in Lompoc, California and Norwalk, Connecticut.

Again teacher training consisted of a general orientation of two initial meetings and monthly meetings thereafter for the purpose of discussion of progress, introduction of new materials, etc.

The program of evaluation included the administration of achievement tests covering each of the books in the series, 1A, 1B, 2A, and 2B. These tests assess the ability of the children in experimental classes to learn material covered in the program and also provide evidence as to specific difficulties and the relative difficulty of particular units of work.

In general, scores on these tests indicate that the content is well within the capacities of first-and second-grade children. Considering the length of the tests the scores are quite high. Table 2 gives the mean of scores on Test 1A and Test 2A.

Table 2

Achievement Tests on Books 1A and 2A, 1961-62

Test	N of children	Total Score Possible	Mean
1A	1803	80	68.44
2A	893	140	120.9

Although considerably more mathematics was presented to and, as evidenced by the tests described, learned by children in experimental classes, the question reasonably is raised "Does the addition of content and emphasis on concepts mean a sacrifice in skills and techniques which have been traditionally the goals of arithmetic teaching?" To determine how well experimental classes can perform on traditional content, standardized achievement tests were administered to a group of randomly selected experimental classes and to matched control classes within the same districts. Classes were matched by administrators within each district on the basis of known variables such as student ability level, socio-economic level of neighborhood, staff capabilities. Children in control classes had been in a traditional program all year. The test was the arithmetic portion of the Metropolitan Achievement Test, Primary I battery (1st grade) and Primary II battery (2nd grade).

Tables 3-7 show results for both groups at each grade and the percentage of each group at specific grade equivalent levels.

Table 3

Grade 1	Exp. Group	Control Group
Number	316	311
Mean Score	58.74	54.96
Median Score	60	56
Range	31-68	23-68

Difference of Means Significant Beyond .001 Level
in Favor of Experimental Group
t value = 5.50

Table 4

Grade 2 Total Test	Exp. Group	Control Group
Number	287	208
Mean Score	55.7	56.0
Median Score	55.8	56.0
Range	30-74	25-74

Difference of Means Not Significant
t value = .38

Table 5

Grade 2 Subtests	Part A: Problem Solving and Concepts		Part B: Computation	
	Exp. Group	Control Group	Exp. Group	Control Group
Mean	55.9	56.0	52.5	53.2
Median	57.5	56.0	49.5	54.0
Range	30-71	27-71	30-71	27-71

Table 6

Grade Equivalent for Grade 1 Children
(Tested at 2.0)

Grade Equivalent	% of Exp. Group	% of Control Group
Above 3.5	23.1	12.5
3.0 - 3.4	25.3	18.3
2.5 - 2.9	21.3	20.5
2.0 - 2.4	23.3	32.7
1.5 - 1.9	6.1	10.0
1.0 - 1.4	.9	5.4
Below 1.0	-	.6

Table 7

Grade Equivalent for Grade 2 Children
(Tested at 3.0)

Grade Equivalent	% of Exp. Group	% of Control Group
Above 4.5	2.8	2.4
4.0 - 4.4	16.7	10.6
3.5 - 3.9	31.6	42.8
3.0 - 3.4	29.6	26.4
2.5 - 2.9	15.3	14.9
2.0 - 2.4	4.0	2.4
1.5 - 1.9	-	-
1.0 - 1.4	-	0.5
Below 1.0	-	-

The standardized test results clearly indicate that children using the Sets and Numbers materials scored at least as well or better on the traditional content as children who had been in traditional programs. In addition, a considerable body of content not tested by standardized tests was taught.

The relatively better results for the experimental group in the first grade appear to the project staff to be due primarily to two factors. One is the greater experience of the first-grade teachers in presenting project material to their classes. The other is the greater emphasis in the second-grade Sets and Numbers books on mathematical content and concepts not examined in the Metropolitan or other standard Achievement Tests. This means that the second grade children in the project spent a relatively larger amount of time learning material on which they were not tested.

5. Ghana Classes - 1962. As a project of the African Education Study of Educational Services Incorporated, eleven classes of Primary One (first grade) children in Accra, Ghana began work in Sets and Numbers, Book 1A at the mid-point of the academic year, 1961-62. Members of the Sets and Numbers project staff were in Accra in February and in May to provide the initial orientation for the classroom teachers and to aid in organizing the testing program at the end of the term. The regular classroom teachers taught the program and the same workbooks used in classes in the United States were used in the Ghanaian classes.

The eleven classes were from five schools in and around Accra. Schools were selected by the Ministry of Education, Ghana, to provide a broad representation in type of background of the students. The program is under the direct supervision of the Ministry of Education.

At the end of the term, approximately two-thirds of the students in each class had completed work in Book 1A. The same achievement test given first-grade experimental classes in the United States at the completion of Book 1A was administered to these children.

Since the upper two-thirds only of each Ghanaian class are represented in the test results, these results were compared with the upper two-thirds of the children in each American class, as shown in Table 8.

Table 8

	Ghanaian classes Administered June, 1962	American classes Administered January, 1962
Sample size	215	1195
Mean	72.4	73.8
S. D.	9.16	8.2

Difference of means is non-significant

6. Experimental Program - 1962-1963. During this academic year, the program has been extended to the third-grade level with Books 3A and 3B of the Sets and Numbers series. In addition a book for the second half of the kindergarten year has been added. Although this book provides a foundation for the work in Book 1A it is not a necessary prerequisite.

The experimental project for 1962-1963 includes:

110 first-grade classes
102 second-grade classes
68 third-grade classes
20 kindergarten classes
50 schools
9 school districts in the United States
approximately 9,000 children.

The project in Ghana includes for 1962-1963:

13 primary one classes
11 primary two classes
7 schools.

The usual teacher training program of orientation meetings and workshops, and meetings throughout the year has been supplemented by a course for the third-grade teachers given at Stanford University, Autumn Quarter, 1962. This was a two-unit course in mathematics for teachers.

Projected plans are for the continued development and testing of materials for grades K through 6.

7. Available Materials. At present the series Sets and Numbers consists of the following workbooks for K through third grade:

K1 for kindergarten

1A and 1B for first grade

2A and 2B for second grade

3A and 3B for third grade.

The first and second grade books have been tested and revised on the basis of test results and teachers' suggestions. The third grade books are presently in experimental form.

There is a teacher's edition for each book in the series, containing background material, comments and suggested procedures for teaching each page, and suggestions for supplementary activities.

Books K1, 1A, 1B, 2A, 2B, and 3A may be ordered from:

L. W. Singer Company
249 West Erie Boulevard
Syracuse 2, New York

At present, the experimental edition of Book 3B may be ordered from:

Sets and Numbers Project
Ventura Hall
Stanford University
Stanford, California

Some published reports of the project are to be found in the following two publications:

Suppes, P. and McKnight, B. "Sets and numbers in grade one, 1959-1960," The Arithmetic Teacher, vol. 8, No. 6 (1961), pp. 287-290;

Suppes, P. and Hill, S. "The concept of set," The Grade Teacher, Vol. LXXIX, No. 8 (1962), pp. 51, 86-90.

A report of a related project is to be found in:

Suppes, P. "Mathematical logic for the schools," The Arithmetic Teacher, vol. 9, No. 7 (1962), pp. 396-399.

8. Related program of psychological research. A program of experimental studies of concept formation in young children is being conducted in close conjunction with the Sets and Numbers project. Particular emphasis has been given to laboratory study of the learning of the kind of mathematical concepts occurring in the workbooks, as for example the concept of identity of sets. This program of psychological research is supported by the Office of Education and the Carnegie Corporation of New York. Reports of the work completed to date may be found in the following publications:

Suppes, P. and Ginsberg, R. "Application of a stimulus sampling model to children's concept formation with and without overt correction responses," Journal of Experimental Psychology, vol. 63, No. 4 (1962), pp. 330-336;

Suppes, P. and Ginsberg, R. "Experimental studies of mathematical concept formation in young children," Science Education, vol. 46, No. 2 (1962), pp. 230-240;

Suppes, P. and Ginsberg, R. "A fundamental property of all-or-none models, binomial distribution of responses prior to conditioning, with application to concept formation in children," Psychological Review, January 1963;

Suppes, P. "Toward a behavioral foundation of mathematical proofs," Technical Report No. 44, Institute for Mathematical Studies in the Social Sciences, Stanford University, January 1962, 23 pp.

9. Staff. The project director is Dr. Patrick Suppes, Professor of Philosophy and Statistics, Stanford University. His research associates are Dr. Shirley Hill, Miss Ann Boyle, Miss Dolly Adams, Miss Catherine Braithwaite, and Mr. Frederick Binford.

10. Project Support. The writing of the workbook material and the accompanying teacher's edition is supported by a grant from the Carnegie Corporation of New York. The program of classroom evaluation and testing is supported by a grant from the national Science Foundation. The project in Ghana is administered through Educational Services Incorporated, Watertown, Massachusetts, and is supported by grants from the Ford Foundation and AID to ESI.

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Stanford, California

M A D I S O N   P R O J E C T   M A T E R I A L S

The following is a list of materials presently available from the Madison Project. Additional units, tapes etc. are in the process of being prepared.

D I S C O V E R Y   I N   A L G E B R A

(workbook and teacher's manual)

The present workbook presents a "first course" in mathematics consisting mainly of fundamental concepts of algebra, arithmetic, and coordinate geometry. The material comprises approximately a two year course when used about once a week. It is intended as enrichment material and does not replace arithmetic.

Discovery in Algebra (workbook and teacher's manual	$9.00
Individual workbooks	3.50

"A X I O M S   F O R   A R I T H M E T I C   A N D   A L G E B R A"

This booklet gives a brief outline of the mathematical content that is involved in the workbook. It is intended as an aid for the Madison Project teacher.

"Axioms for Arithmetic and Algebra"	$1.80

"M A T R I C E S, F U N C T I O N S, A N D   O T H E R   T O P I C S"

This volume was written to follow the book Discovery in Algebra. Although this material can be used alone, a familiarity with some of the mathematical topics presented in Discovery in Algebra are necessary to the studies presented in this volume.

Matrices, Functions, and Other Topics (workbook)	$1.00
Matrices,Functions, and Other Topics (Teacher's manual)	$1.00

M A D I S O N   P R O J E C T   F I L M S

The following are the films that are presently available from the Project:

- A Lesson with Second Graders
- Graphing and Ellipse
- Matrices
- Complex Numbers via Matrices
- Derivations
- First Lesson
- Second Lesson
- Sequencing
- Discovery

TAPE RECORDINGS

Tape #D-1, showing a good class of fifth grade students at Weston, Connecticut proving algebraic theorems from a set of axioms.
Price: $2.50

Tape #D-2, showing a good class of fourth graders at Weston, Connecticut in a lesson on: quadratic equations with signed number roots, proof of an algebraic theorem from a set of axioms, formulas for solution of equations with literal coefficients.
Price: $3.50

Tape #D-3, using an average class from Scarsdale, New York in a first lesson on: open sentences, truth sets, identities, rule for substitution, the pet store game and the matrix game.
Price $3.50

Any two of the above tapes may be rented for $1.50. All three may be rented for $2.00. The tapes may be copied if you so desire.

Additional information concerning the films and requests for securing the films should be directed to:

Madison Project Office
Webster College
470 Lockwood Avenue
Webster Groves 19, Missouri

Additional information and orders for other materials should be directed to:

Madison Project
Mathematics Department
Syracuse University
Syracuse 10, New York