

3<sup>a</sup> parte (4)

	0	1	2	3	4	5	6	7	8	9	10	2	2	7d	fd <sup>2</sup>
0	1 (20) 20	2 (32) 12	1 (12) 12	4 (32) 8		3 11		4 (36) -8		2 (36) -16		17	-4	-68	272
1	1 (15) 15	2 (24) 12										3	-3	-9	27
2	1 (10) 10	2 (16) 8	1 (6) 6	1 (4) 4		2 11		1 (4) -4				7	-2	-14	28
3	1 (5) 5	3 (12) 4				1 11		2 (4) -2				7	-1	-7	7
4			2 (4) 4			1 11				4 11		21	-	0	0
5	5 (25) -5	2 (8) -4	4 (12) -3	4 (8) -2		4 11		8 (16) -2		5 (20) -5	1 (6) -5	33	+1	33	33
6												-	2	0	0
7			3 (27) -9	6 (36) -6		11 11		10 (60) -6		9 (108) -12	4 (60) -15	43	3	129	387
7	9	12	11	19	-	22	-	33	-	20	5	131		Σ=64	Σ=754
8	-5	-7	-3	-2	-1	-	1	2	3	4	5			Σ u <sub>y</sub> = 259	
7d	-45	-48	-33	-32	0	0	0	66	0	80	25	Σ=7		+ 447 - 188 259	
7d <sup>2</sup>	225	172	99	76	0	0	0	132	0	320	125	Σ=1.163			

1<sup>a</sup> parte (a)

$$M = M_0 + \frac{\sum df}{\sum f} \cdot i$$

$$M = 5 + \frac{7}{131} \cdot i$$

$$M = 5 + 0,05 \cdot i = 5,1$$

$$\sigma = i \sqrt{\frac{\sum f d^2}{\sum f} - \left(\frac{\sum f d}{\sum f}\right)^2}$$

$$\sigma = \sqrt{\frac{1.169}{131} - \left(\frac{7}{131}\right)^2}$$

$$\sigma = \sqrt{8,92 - 0,0025}$$

$$\sigma = \sqrt{8,9175}$$

$$\sigma = 2,98$$

u	7	d	fd
0	7	-5	-35
1	12	-4	-48
2	11	-3	-33
3	19	-2	-38
4	-	-1	0
5	22	0	0
6	-	1	0
7	33	2	66
8	-	3	0
9	20	4	80
10	5	5	25
	Σ=131		Σ=7

$$M = M_0 + \frac{\sum df}{\sum f} \cdot i$$

$$M = 3 + \frac{195}{131} \cdot i$$

$$M = 3 + 1,5 = 4,5$$

$$\sigma = i \sqrt{\frac{\sum f d^2}{\sum f} - \left(\frac{\sum f d}{\sum f}\right)^2}$$

$$\sigma = \sqrt{\frac{754}{131} - \left(\frac{64}{131}\right)^2}$$

$$\sigma = \sqrt{5,75 - 0,2304}$$

$$\sigma = \sqrt{5,5196}$$

$$\sigma = 2,34$$

$$r = \frac{\sum u_y - c_u c_y}{\sigma_u \sigma_y} \cdot i$$

$$r = \frac{259}{131} - 0,48 \times 0,05$$

$$r = \frac{2,34 \times 2,98}{2,34 \times 2,98}$$

$$r = \frac{1,97 - 0,0240}{6,9732}$$

$$r = \frac{1,946}{6,9732}$$

$$r = 0,28$$

$$\sigma_r = \frac{1 - r^2}{\sqrt{N}}$$

$$\sigma_r = \frac{1 - 0,0784}{11,44}$$

$$\sigma_r = \frac{0,9216}{11,44}$$

$$\sigma_r = 0,08$$

$$r \pm 3\sigma_r$$

$$0,04 \leftarrow \rightarrow 0,52$$

u	7	d	fd
0	12	-3	-36
1	3	-2	-6
2	7	-1	-7
3	7	2	14
4	21	1	21
5	33	2	66
6	-	3	0
7	43	4	172
	Σ=131		Σ=195

all



4<sup>a</sup> parte (y)

	0	1	2	3	4	5	6	7	8	9	10	Σ	d	d <sup>2</sup>	yd	yd <sup>2</sup>
0	20 11 2	16 10 1	12 8 2		7 3 2	-4 -2 2	-7 -16 2	-12 -20 5	-16 -20 5	-20 -40 2		131	-4	16	72	288
1					3 1 1	3 -3 1						21	-3	9	-6	18
2		8 1 1			2 1 1	2 -2 1	-4 -12 3			-2 -2 1		7	-2	4	-14	28
3	5 1 1		3 1 1		1 1 1	1 -1 1	-2 -2 1					7	-1	1	-7	7
4												21	0	0	0	0
5	-5 11 4	-1 12 3	-3 12 6		-1 11 4	1 11 6	2 11 4	4 1 1	5 1 1	10 1 1		33	1	1	33	33
6												-	2	4	0	0
7	-15 1 1	-12 12 1	-7 12 0		-3 12 4	3 12 6	6 12 4	12 12 2	15 12 2	30 1 1		43	3	9	129	387
Σ	9	8	23	0	15	15	19	20	0	16	6	131			Σ = 63	Σ = 761
d	-5	-4	-3	-2	-1	0	1	2	3	4	5				Σ uy = 15	
yd	-45	-32	-69	0	-15	0	19	40	0	64	30				Σ = -8	
yd <sup>2</sup>	225	128	207	0	15	0	19	80	0	256	150				Σ = 1080	

1<sup>a</sup> parte (x)

u	7	d	d <sup>2</sup>
0	4	-5	25
1	8	-4	16
2	23	-3	9
3	0	-2	4
4	15	-1	1
5	15	0	0
6	19	1	1
7	20	2	4
8	0	3	9
9	10	4	16
10	6	5	25

N=131  
Σ = -8

$M = M_0 + \frac{\sum df}{\sum f} i$   
 $M = 5 + \frac{-8}{131}$   
 $M = 5 + 0,06 = 5,1$   
 $\sigma = \sqrt{\frac{\sum x^2}{\sum f} - \left(\frac{\sum df}{\sum f}\right)^2}$   
 $\sigma = \sqrt{\frac{10,80}{131} - \left(\frac{-8}{131}\right)^2}$   
 $\sigma = \sqrt{8,24 - 0,0036}$   
 $\sigma = \sqrt{8,2364}$   
 $\sigma = 2,86$

u	7	d	d <sup>2</sup>
0	18	-4	16
1	2	-3	9
2	7	-2	4
3	7	-1	1
4	21	0	0
5	33	1	1
6	0	2	4
7	43	3	9

N=131  
Σ = 63

$M = M_0 + \frac{\sum df}{\sum f} i$   
 $M = 4 + \frac{63}{131}$   
 $M = 4 + 0,5 = 4,5$   
 $\sigma = \sqrt{\frac{\sum x^2}{\sum f} - \left(\frac{\sum df}{\sum f}\right)^2}$   
 $\sigma = \sqrt{\frac{761}{131} - \left(\frac{63}{131}\right)^2}$   
 $\sigma = \sqrt{5,8 - 0,2304}$   
 $\sigma = \sqrt{5,5696}$   
 $\sigma = 2,36$

$r = \frac{\sum uy - c_u c_y}{\sigma_u \sigma_y} \dots i$

$r = \frac{15}{131} - 0,06 \times 0,44$   
 $r = \frac{15}{131} - 0,0264$   
 $r = 0,114 - 0,0264$   
 $r = 0,0876$

$r = \frac{0,114 - 0,0264}{0,7476}$

$r = \frac{0,0876}{0,7476}$

$r = 0,01$

$\sigma_r = \frac{1 - r^2}{\sqrt{N}}$

$\sigma_r = \frac{1 - 0,0001}{11,44}$

$\sigma_r = \frac{0,9999}{11,44}$

$\sigma_r = 0,08$

r ± 3σ<sub>r</sub>



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$3^2$  factorial (d)

	0	1	2	3	4	5	6	7	8	9	10	$\bar{y}$	$\bar{y}^2$	$\bar{y}^3$	$\bar{y}^4$
0	60		50	80						10	10	8	-4	-32	128
1		50		60				40		10	10	8	-3	-27	72
2	10		10	70				40		10	10	8	-2	-16	38
3	5	10	5	10				20		10	10	36	-1	-36	36
4												0	0	0	0
5		10	5	10				10		10	10	22	1	22	22
6												0	2	0	0
7				50				10		10	10	26	3	72	234
8												0	4	0	0
9	10	10	10					10		10	10	12	5	60	300
10								10		10	10	5	6	30	180
$\bar{y}$	3	12	-11	19	0	22	0	33	0	20	5	131		$\Sigma x$ 28	$\Sigma x^2$ 1010
d	.5	-.4	.3	-.2	-.1	0	1	2	3	4	5			$\Sigma u_y = 322$	
$\bar{y}^2$	-45	-42	-33	-32	-0	0	0	66	0	20	25			+ 534 - 215 322	
$\bar{y}^3$	225	192	99	76	0	0	0	138	0	320	125			$\Sigma u_y^2$ 1169	

u	v	w	uv
0	1	-5	-40
1	1	-4	-32
2	2	-3	-24
3	36	-2	-72
4	0	-1	-18
5	22	0	0
6	0	1	0
7	26	2	52
8	0	3	0
9	12	4	48
10	5	5	25
	131		$\Sigma = -49$

$$\mu = \bar{y} + \frac{\Sigma uv}{\Sigma v}$$

$$\mu = 5 + \frac{-49}{131}$$

$$\mu = 5 - 0,37$$

$$\mu = 4,63$$

$$\sigma = \sqrt{\frac{\Sigma uv^2}{\Sigma v} - \left(\frac{\Sigma uv}{\Sigma v}\right)^2}$$

$$\sigma = \sqrt{\frac{1010}{131} - \left(\frac{-49}{131}\right)^2}$$

$$\sigma = \sqrt{7,71 - 0,1429}$$

$$\sigma = \sqrt{7,5671}$$

$$\sigma = 2,77$$

u	v	w	uv
0	9	-5	-45
1	12	-4	-48
2	11	-3	-33
3	19	-2	-38
4	0	-1	0
5	22	0	0
6	0	1	0
7	33	2	66
8	0	3	0
9	20	4	80
10	5	5	25
			$\Sigma = 7$

$$\mu = \frac{\Sigma uv}{\Sigma v} = \frac{7}{131}$$

$$\sigma = \sqrt{\frac{\Sigma uv^2}{\Sigma v} - \left(\frac{\Sigma uv}{\Sigma v}\right)^2}$$

$$\sigma = \sqrt{\frac{322}{131} - \left(\frac{7}{131}\right)^2}$$

$$\sigma = \sqrt{2,45 - 0,0335}$$

$$\sigma = \frac{2,4165}{8,0162}$$

$$\sigma = 0,301$$

$$\sigma_1 = \frac{1 - \mu^2}{\sqrt{N}}$$

$$\sigma_1 = \frac{1 - 0,0022}{11,44}$$

$$\sigma_1 = \frac{0,9978}{11,44}$$

$$\sigma_1 = 0,087$$

$$\mu = 0,08 \leftarrow \rightarrow 0,51$$

3<sup>2</sup> factorial (d)



5<sup>a</sup> parte (y)

	0	1	2	3	4	5	6	7	8	9	10	7	12	72	72 <sup>2</sup>
0	16	12					-8		16	20	24	12	-4	-68	272
1	12	9							15			3	-3	-9	27
2	8	6					-4				12	7	-2	-14	28
3	4	3				-1		-3				7	-1	-7	7
4												21	0	0	0
5	4	3	2					3	4	5	6	33	1	33	33
6												2	0	0	
7	12	9	6	3		3	6	9	12	15	18	43	3	129	387
7	59	16	5	4	5	2	5	2	5	16	7	131	$\sum z = 64$	$\sum z^2 = 754$	
8	-4	-3	-2	-1	0	1	2	3	4	5	6		$\sum z y = -17$		
72	236	-48	-10	-4	0	2	10	21	20	80	42		$\sum z = -482$	$+403$	$-17$
72 <sup>2</sup>	944	144	20	4	0	2	20	63	80	400	252		$\sum z = 1729$		

1<sup>a</sup> parte (x)

0	59	-8	-293
1	16	-4	-64
2	8	-2	-18
3	4	-1	-8
4	3	-1	-3
5	2	0	0
6	3	1	3
7	3	2	14
8	5	3	18
9	16	7	64
10	7	5	245
	131		357

$M = M_0 + \frac{\sum x_i}{n}$   
 $M = 5 + \frac{-293}{131}$   
 $M = 5 - 2,23$   
 $M = 2,77$   
 $\sigma = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$   
 $\sigma = \sqrt{\frac{1729}{131} - \left(\frac{-293}{131}\right)^2}$   
 $\sigma = \sqrt{13,25 - 5,2077}$   
 $\sigma = \sqrt{8,0423}$   
 $\sigma = 2,84$

x	f	d	fd
0	12	-4	-68
1	3	-3	-9
2	7	-2	-14
3	7	-1	-7
4	21	0	0
5	33	1	33
6	-	2	0
7	43	3	129
	131		207

$M = M_0 + \frac{\sum fd}{\sum f}$   
 $M = 5 + \frac{207}{131}$   
 $M = 5 + 1,58$   
 $M = 6,58$   
 $\sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$   
 $\sigma = \sqrt{\frac{754}{131} - \left(\frac{207}{131}\right)^2}$   
 $\sigma = \sqrt{5,75 - 0,2304}$   
 $\sigma = \sqrt{5,5196}$   
 $\sigma = 2,34$

$z = \frac{\frac{\sum x_i}{n} - C_x C_y}{\sigma_x \sigma_y}$   
 $z = \frac{-17}{131} = -0,1298$   
 $z = \frac{-0,14 - 0,44 \cdot 4}{2,34 \cdot 2,72}$   
 $z = \frac{-0,5864}{6,3648}$   
 $z = -0,09$

-0,20 - 0,248

$\sigma_x = \frac{1 - z^2}{\sqrt{N}}$   
 $\sigma_x = \frac{1 - 0,0081}{\sqrt{131}}$   
 $\sigma_x = \frac{0,9919}{11,44}$   
 $\sigma_x = 0,0867$   
 $\sigma_y = 0,08$   
 $\sigma_{xy} = 0,07$



	0	1	2	3	4	5	6	7	8	9	10	f	d	fd	fd <sup>2</sup>
0	50			-10	-15			-30	-35			9	-5	-45	225
1	40	12				-16	-20		-28			12	-4	-48	192
2	24	6					-15	-18		-21	-24	10	-3	-30	90
3	24	6					24	10	36	28	16	20	-2	-40	80
4		162											-1		
5															22
6															
7	18	4		4	8		8	20	56	16		33	2	66	132
8													3		
9	22	8		4	16	12		24	84	32		20	4	80	320
10	20	5						30	35	40		5	5	25	125
f	59	17	5	4	5	2	5	7	5	16	6	131		8	1164
d	-2	-1		1	2	3	4	5	6	7	8				
fd	-118	-17	135	4	10	6	20	35	30	112	48	130			
fd <sup>2</sup>	236	17		4	20	18	80	175	180	784	384	1898			

$$\sigma_x = \frac{1898}{131} - \left(\frac{130}{131}\right)^2 = \frac{1898 \times 131 - 16900}{131}$$

$$\sigma_x = \frac{248638 - 16900}{131} = \frac{231738}{131}$$

$$\sigma_x = \frac{481.32}{131} = 3.67$$

$$\sigma_y = \frac{\sqrt{1164 \times 131 - 64}}{131} = \frac{\sqrt{152484 - 64}}{131}$$

$$\sigma_y = \frac{\sqrt{152420}}{131} = \frac{390.71}{131} = 2.99$$

$$r = \frac{\frac{1}{131} - \frac{8}{131} \times \frac{130}{131}}{2.99 \times 3.67}$$

$$r = \frac{0.04}{131} - \frac{1040}{17161}$$

$$r = \frac{131}{17161} - \frac{1040}{17161} = \frac{909}{17161}$$

$$r = \frac{-0.05296}{10.9636} = -0.0048$$

$$\sigma_r = \frac{1 - r^2}{\sqrt{131}}$$

$$\sigma_r = \frac{1 - 0.00002304}{11.44}$$

$$\sigma_r = \frac{0.99997696}{11.44}$$

$$\sigma_r = 0.087$$

$$3\sigma_r = 0.261$$

-0.2658 ——— 0.26052



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	0	1	2	3	4	5	6	7	8	9	10	f	d	fd	fd <sup>2</sup>
0	3	5							6		2	2	-4	-32	128
1	2	2	3				159				8	8	-3	-24	72
2	1	4	1					4	6	3	9	8	-2	-16	32
3	2	2	2			2	4	6	4	2	2	35	-1	-35	35
4															
5	1	1						2	3	5	6	28	1	28	28
6							187						2	-	
7	1	1	3					6	9	12	15	24	3	81	243
8													4	-	
9	1	1								2	2	12	5	60	300
10	1	1										5	6	30	180
f	59	16	5	4	5	2	5	7	5	16	7	131		92	1018
d	-3	-2	-1		1	2	3	4	5	6	7				
fd	-177	-32	-5		5	4	15	28	25	96	49	8			
fd <sup>2</sup>	531	64	5		5	8	45	112	125	576	343	1814			

$$\sigma_x = \frac{\sqrt{1814 \times 181 - 92^2}}{131} = \frac{\sqrt{237634 - 64}}{131} = \frac{\sqrt{237570}}{131} = \frac{487.4}{131} = 3.72$$

$$\sigma_y = \frac{\sqrt{1018 \times 131 - 92^2}}{131} = \frac{\sqrt{133358 - 8464}}{131} = \frac{\sqrt{124894}}{131} = \frac{353.2}{131} = 2.69$$

$$r = \frac{-\frac{211}{131} - \frac{92}{131} \times \frac{8}{131}}{3.72 \times 2.69} = \frac{-\frac{211}{131} - \frac{736}{17161}}{10.0068} = \frac{-27641 - 736}{17161} = \frac{-28377}{17161} = \frac{-1.6535}{10.0068}$$

$$r = -0.165$$

$$\sigma_r = \frac{1 - 0.027225}{11.44} = \frac{0.972775}{11.44}$$

$$\sigma_r = 0.085$$

$$3 \sigma_r = 0.255$$





2ª parte

	0	1	2	3	4	5	6	7	8	9	10	f	x	fx	fx <sup>2</sup>
0	III +16 3 (+8)	III +16 3 (+8)	III +16 6 (+24)				III -16 3 (-6)			III -20 2 (-10)		17	14	-62	272
1		II -9 2 (-4.5)	II -9 4 (-18)	II -9 3 (-27)								2	3	-6	12
2		II -6 2 (-3)	II -6 4 (-12)	II -6 3 (-18)						I -10 1 (-10)		4	7	-14	28
3		I +3 1 (+3)	I +3 2 (+6)	I +3 3 (+9)								8	1	8	8
4	I	II	III	IIII								20			
5	II -4 2 (-2)	I -3 1 (-1.5)	II -2 2 (-1)	IIII -1 2 (-0.5)						IIII +5 2 (+2.5)	IIII +26 2 (+13)	34	1	34	34
6													2		
7	II -12 2 (-6)	II -6 2 (-3)	IIII +7 2 (+3.5)							IIII -15 3 (-5)	IIII -18 3 (-6)	43	3	129	337
f	8	10	7	35		28		26		120	5	137		67	747
d	-4	-3	-2	-1		1	2	3	4	5	6				
fd	-32	-30	-14	-35		28		72		60	30	85			
fd <sup>2</sup>	128	90	28	35		28		234		300	180	1023			

1ª parte (up)

M = 4,51

σ = 2,33

$$\sigma = i \sqrt{\frac{\sum fd^2}{f} - \left(\frac{\sum fd}{f}\right)^2} = \sigma = \sqrt{\frac{747}{131} - \left(\frac{67}{131}\right)^2} = \sqrt{5,7 - 0,2601} =$$

$$\sqrt{5,4399} = 2,33$$

$$r = \frac{\sum xy - c_x c_y}{\sigma_x \sigma_y} \quad ii$$

$$r = \frac{284}{131} - 0,65 \times 0,51$$

$$r = \frac{2,72 \times 2,33}{2,72 \times 2,33}$$

$$r = \frac{2,16 - 0,3315}{6,3376}$$

$$r = \frac{1,8285}{6,3376}$$

$$r = 0,28$$

$$\sigma_r = \frac{1 - r^2}{\sqrt{N}}$$

$$\sigma_r = \frac{1 - (0,28)^2}{\sqrt{131}} = \frac{1 - 0,0774}{11,44} = \frac{0,9226}{11,44} = 0,083$$

$$3\sigma_r = 0,249$$

$$0,031 \leftarrow \rightarrow 0,529$$

2ª parte X

M = 4,65

σ = 2,72

$$\sigma = i \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

$$\sigma = \sqrt{\frac{1023}{131} - \frac{7235}{131^2}}$$

$$\sigma = \sqrt{\frac{1023 \times 131 - 7235}{131^2}}$$

$$\sigma = \sqrt{\frac{126788}{131^2}}$$

$$\sigma = \frac{356}{131}$$

σ =

f.d.



5<sup>a</sup> parte

	0	1	2	3	4	5	6	7	8	9	10	f	d	fd	fd <sup>2</sup>
0	12 108 111 9		237									9	-4	-36	144
1	45 45 5 1	6 6 1							-15 -15 1			8	-3	-24	72
2	66 66 4 1	2 2 1	14 14 2		-2 -2 1		-6 -6 2	-8 -8 2	-12 -12 1	-14 -14 1		23	-2	-46	92
3													-1		
4												15		-106	230
5	30 30 3 1	4 4 2		1 1 1			4 4 1	5 5 1	18 18 3			15	1	15	15
6	30 30 3 1	4 4 2	10 10 2	2 2 1				12 12 2	14 14 1			17	2	38	76
7	55 55 4 1	14 14 2				3 3 1	12 12 2	15 15 3	18 18 1	21 21 1		19	3	57	171
8													4		
9	15 15 3 1	4 4 2				20 20 1	25 25 2	30 30 3	35 35 1	40 40 4		17	5	85	425
10	15 15 3 1	6 6 2					20 20 2					6	6	36	216
f	59	16	5	4	5	2	5	8	5	15	7	131		124	1311
d	-3	-2	-1		1	2	3	4	5	6	7				
fd	-18	-32	-5		5	10	20	30	40	90	49	65			
fd <sup>2</sup>	354	64	5		5	8	45	128	125	540	393	1617			

$M = M_0 + \frac{\sum fd}{N} \cdot i$

u	f	d	fd
0	9	-5	-45
1	8	-4	-32
2	23	-3	-69
3	15	-2	-30
4	15	-1	-15
5	15	0	0
6	19	1	19
7	19	2	38
8	17	3	51
9	17	4	68
10	6	5	30
	131		155

$M = 5 - \frac{6}{131}$   
 $M = 5 - 0.046$   
 $M = 4.95$

$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$   
 $\sigma = \sqrt{\frac{1311}{131} - \left(\frac{155}{131}\right)^2}$   
 $\sigma = \sqrt{10.0076 - 1.376}$   
 $\sigma = \sqrt{8.631}$   
 $\sigma = 2.94$   
 $\sigma = 2.88$

5<sup>a</sup> parte

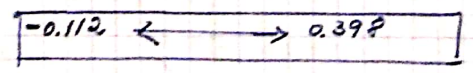
u	f	d	fd
0	59	-4	-236
1	16	-2	-32
2	5	-2	-10
3	4	-1	-4
4	5	0	0
5	2	1	2
6	2	2	4
7	2	3	6
8	15	4	60
9	7	5	35
10	7	6	42
	129		125

$M = M_0 + \frac{\sum fd}{N} \cdot i$   
 $M = 4 - \frac{125}{131}$   
 $M = 4 - 0.95$   
 $M = 3.05$

$\sigma = \sqrt{\frac{1617 \times 131 - 65^2}{131}}$   
 $\sigma = \sqrt{\frac{211827 - 4225}{131}}$   
 $\sigma = \sqrt{1607.602}$   
 $\sigma = 40.063$   
 $\sigma = 3.47$

$r = \frac{\frac{\sum xy}{N} - \bar{x} \cdot \bar{y}}{\sigma_x \sigma_y} (i_x \cdot i_y)$   
 $r = \frac{\frac{246}{131} - (0.4 \times 0.9)}{2.88 \times 3.47}$   
 $r = \frac{1.88 - 0.45}{10}$   
 $r = \frac{1.43}{10}$   
 $r = 0.143$

$\sigma_r = \frac{1 - r^2}{\sqrt{N}}$   
 $\sigma_r = \frac{1 - (0.14)^2}{\sqrt{131}}$   
 $\sigma_r = \frac{1 - 0.0196}{11.44}$   
 $\sigma_r = 0.085$   
 $3\sigma_r = 0.255$





4ª parte

	0	1	2	3	4	5	6	7	8	9	10	f	d	fd	fd²
0	$\frac{20}{2}$ 40	$\frac{15}{1}$ 15	$\frac{10}{3}$ 30	-	-	-	-	-	-	$\frac{-25}{1}$ -25	$\frac{-30}{1}$ -30	9	-5	-45	225
1	$\frac{16}{2}$ 32	$\frac{12}{1}$ 12	$\frac{8}{2}$ 16	-	-	$\frac{-4}{2}$ -8	$\frac{-8}{1}$ -8	$\frac{-12}{1}$ -12	-	-	-	12	-4	-48	192
2	$\frac{12}{1}$ 12	-	$\frac{6}{3}$ 18	-	-	$\frac{-3}{4}$ -12	$\frac{-6}{1}$ -6	$\frac{-9}{1}$ -9	-	-	$\frac{-18}{1}$ -18	11	-3	-33	99
3	$\frac{8}{1}$ 8	$\frac{6}{3}$ 18	$\frac{4}{4}$ 16	-	-	$\frac{-2}{1}$ -2	$\frac{-4}{4}$ -16	$\frac{-6}{2}$ -12	-	-	$\frac{-10}{2}$ -20	19	-2	-38	76
4	-	-	-	-	-	-	-	-	-	-	-	-	-1	-	-
5	2	1	1	-	1	3	2	8	-	3	1	22	-	-	164 171
6	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-
7	$\frac{-8}{1}$ -8	$\frac{-6}{1}$ -6	$\frac{-4}{7}$ -28	-	4	$\frac{4}{2}$ 2	$\frac{36}{9}$ 4	$\frac{36}{6}$ 6	-	$\frac{10}{2}$ 20	$\frac{12}{1}$ 12	33	2	66	132
8	-	-	-	-	-	-	-	-	-	-	-	-	3	-	-
9	-	$\frac{-12}{1}$ -12	$\frac{-8}{3}$ -24	-	3	$\frac{4}{2}$ 2	$\frac{8}{2}$ 4	$\frac{12}{2}$ 6	-	$\frac{20}{5}$ 4	$\frac{24}{2}$ 12	20	4	80	320
10	-	-	-	-	5	$\frac{5}{1}$ 5	-	-	-	$\frac{25}{3}$ 25	$\frac{25}{3}$ 25	5	5	25	125
f	9	8	23	-	15	15	19	20	-	16	6	131	-	7	1169
d	-4	-3	-2	-1	-	1	2	3	4	5	6	-	-	-	-
fd	-36	-24	-46	-	106 229	15	38	60	-	80	36	113	-	-	-
fd²	144	72	92	-	-	15	76	180	-	400	216	1195	-	-	-

3ª parte:  
M = 5.06  
σ = 2.98

4ª parte:  
M = 4.94  
σ = 2.05

$$r = \frac{\sum xy - c_x c_y}{N \sigma_x \sigma_y}$$

$$r = \frac{\frac{345}{131} - \frac{7}{131} \times \frac{113}{131}}{2.98 \times 2.05}$$

$$r = \frac{\frac{345}{131} - \frac{791}{131^2}}{6,1090} = \frac{\frac{345-6}{131}}{6,1090} = \frac{\frac{339}{131}}{6,1090} = \frac{2,5878}{6,1090} =$$

$$r = 0.42$$

$$\sigma_r = \frac{1-r^2}{\sqrt{N}}$$

$$\sigma_r = \frac{1-0.1764}{\sqrt{131}} = \frac{0,8236}{11,44} =$$

$$\sigma_r = 0.072$$

$$3\sigma_r = 0.216$$

$$r \rightarrow 0.636 \leftarrow 0.204$$



4a

	0	1	2	3	4	5	6	7	8	9	10	f	d	fd	fd <sup>2</sup>
0	16 16 1		8 24 3							-20 40 2	-24 48 2	8	-4	-32	128
1	12 12 1	9 9 1	6			-3 3 1	-6 18 3	-9 9 1				8	-3	-24	72
2	8	6 12 2	4 12 3			-2 2 1	-1	-6 6 1				7	-2	-14	28
3	4 16 4	3 3 1	2 16 8	120		-1 1 1	-2 2 1	-3 3 1		-5 30 6		35	-1	-36	36
4														-106 198	92
5		3 6 2	2 10 5	-132		1 2 1	2 8 4	3 6 3		5 10 2	6 6 1	28	1	28	28
6							4 4 1					1	2	2	4
7	12 24 2	9 9 1	6 18 3			3 3 1	6 30 5	9 27 3		15 75 5	18 18 1	26	3	78	234
8															
9	16 16 1	15 15 1	10 10 1			5 5 1	10 10 1	15 30 2		25 25 1	30 30 1	12	5	60	300
10						6 18 3		18 18 1			36 36 1	5	6	30	180
f	9	8	23		15	15	19	20		16	6	131		92	1040
d	-4	-3	-2	-1		1	2	3	4	5	6				
fd	-36	-24	-46		-106 229	15	38	60		80	36	123			
fd <sup>2</sup>	144	72	92			15	76	180		400	196	1175			

2a

$$\sigma_x = \frac{\sqrt{1010 \times 131 - 92^2}}{131} = \frac{\sqrt{132310 - 8464}}{131} = \frac{\sqrt{123846}}{131} = \frac{366}{131}$$

$$\sigma_x = 2.79$$

$$\sigma_y = \frac{\sqrt{1175 \times 131 - 123^2}}{131} = \frac{\sqrt{153925 - 15129}}{131} = \frac{\sqrt{138796}}{131} = \frac{372.56}{131}$$

$$\sigma_y = 2.84$$

$$r = \frac{\frac{\sum xy}{N} - \sigma_x \sigma_y}{\sigma_x \sigma_y}$$

$$r = \frac{\frac{197}{131} - \frac{123}{131} \times \frac{92}{131}}{\frac{2.79 \times 2.84}{17161}} = \frac{\frac{25804}{17161} - \frac{11316}{17161}}{7.9236} =$$

$$r = \frac{14491}{17161} = \frac{0.84409}{7.9236} = 0.106$$

$$\sigma_r = \frac{1 - 0.0112361}{11.44} = \frac{0.988764}{11.44} = 0.0864$$

$$3\sigma_r = 0.2592$$

$$[-0.153 \quad \text{---} \quad 0.365]$$

8.



Table (4)

	0-3	4-7	8-11	12-15	16-19	20-23	24-27	28-31	32-35	36-39	40-43	f	d	fd	fd <sup>2</sup>
0		1 <sub>3</sub> (4)		8 <sub>3</sub> (2)	4 <sub>4</sub> (16)			8 <sub>1</sub> (8)	12 <sub>1</sub> (12)			17	-4	-68	272
1			3 <sub>2</sub> (12)									3	-3	-9	27
2			6 <sub>2</sub> (12)		2 <sub>1</sub> (2)			2 <sub>3</sub> (6)				7	-2	-12	28
3		4 <sub>1</sub> (4)		2 <sub>3</sub> (6)	1 <sub>2</sub> (2)			2 <sub>1</sub> (2)				7	-1	-7	7
4		1 <sub>7</sub> (14)	1 <sub>3</sub> (12)	1 <sub>5</sub> (12)	1 <sub>5</sub> (12)	1 <sub>5</sub> (12)	1 <sub>4</sub> (12)	2 <sub>2</sub> (12)	3 <sub>1</sub> (12)			21	0	0	0
5		4 <sub>1</sub> (14)	3 <sub>4</sub> (12)	4 <sub>3</sub> (12)	1 <sub>2</sub> (2)		1 <sub>5</sub> (12)	2 <sub>4</sub> (12)	3 <sub>3</sub> (12)			33	1	33	33
6												0	2	0	0
7				5 <sub>2</sub> (10)	3 <sub>4</sub> (12)		3 <sub>3</sub> (12)	6 <sub>2</sub> (12)	9 <sub>1</sub> (9)	12 <sub>1</sub> (12)	15 <sub>2</sub> (30)	43	3	129	387
8															
9															
10															
f	6	8	14	18	25	21	21	12	4	2	131	$\Sigma = 66$			$\Sigma = 750$
d	4	-3	-2	-1	0	1	2	3	4	5				$\Sigma uy = 324$	
fd	-24	-24	-28	-18	0	21	42	36	16	10	$\Sigma = 31$			$+ 400$ $- 76$ $324$	
fd <sup>2</sup>	96	72	56	18	0	21	84	108	64	50	$\Sigma = 569$				

f: points (2)

$$\sigma = \sqrt{\frac{\Sigma d^2 f}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$\sigma = \sqrt{\frac{750}{131} - \left(\frac{66}{131}\right)^2}$$

$$\sigma = \sqrt{5,72 - 0,25}$$

$$\sigma = \sqrt{5,47}$$

$$\sigma = 2,3$$

y

$$\sigma = 4 \sqrt{\frac{569}{131} - \left(\frac{31}{131}\right)^2}$$

$$\sigma = 4 \sqrt{4,34 - 0,0529}$$

$$\sigma = 4 \sqrt{4,2871}$$

$$\sigma = 4 \times 2,07$$

$$\sigma = 8,28$$

$M_u = 4,5$   
 $M_y = 20,23$

$$r = \frac{\frac{324}{131} - 0,5 \times 0,23}{2,3 \times 2,07}$$

$$r = \frac{2,473 - 0,115}{4,761}$$

$$r = \frac{2,358}{4,761}$$

$$r = 0,49$$

$$\sigma_r = \frac{1-r^2}{\sqrt{N}}$$

$$\sigma_r = \frac{1-0,2401}{11,47}$$

$$\sigma_r = \frac{0,7599}{11,47}$$

$$\sigma_r = 0,06$$

$$362 = 0,18$$

$$\boxed{0,31 \quad \quad \quad 0,62}$$



Total

	0	3	4	7	8	11	12	15	16	19	20	23	24	27	28	31	32	35	36	39	40	43	f	d	fd	fd <sup>2</sup>				
0			16	12	8	4								-4									8	-4	-32	128				
1			12	9	6																		8	-3	-24	72				
2			6	4	2									-2	-4								8	-2	-16	32				
3			4	3	2	1								-1	-2	-3							36	-1	-36	36				
4																									-108	196				
5														1	2	3							28	1	28	28				
6																									2					
7																										26	3	78	234	
8																											4			
9																											12	5	60	300
10																											5	6	30	180
f		6	8	14	18	25	21	21	12	4	2											131		88	1010					
d		-4	-6	-2	-1									1	2	3	4	5												
fd		-24	-24	-28	-18									21	42	36	16	10				31								
fd <sup>2</sup>		96	72	56	18									21	84	108	64	50				569								

2<sup>nd</sup> p.  $\sigma = 2.69$

Total  $\sigma = 2.07 \times 4$

$$r = \frac{365}{131} - \frac{31 \times 88}{17161}$$

$$2.59 \times 2.07$$

$$47815 - 2.228$$

$$r = \frac{17161}{5.5693} = \frac{2,6287}{5,5683} = 0.47$$

$$\sigma_r = \frac{1 - 0.2209}{11.44} = \frac{0.7791}{11.44} = 0.068$$

$$3 \sigma_r = 0.20$$

$$0.27 \leftarrow \rightarrow 0.67$$



total (4)

	0-3	4-7	8-11	12-15	16-19	20-23	24-27	28-31	32-35	36-39	40-43	f	d	fd	fd <sup>2</sup>
0		20 40 2	15 30 2	10 20 2	5 10 2	1						9	-5	-45	225
1		16 32 3	12 24 3	8 16 3	4 8 3	3						12	-4	-48	192
2			7 14 2	6 12 1	3 6 4	2 4 1	3 6 1					11	-3	-33	99
3				2 4 4	3 6 3	2 4 4	3 6 2					19	-2	-38	76
4												φ	-1	-4	4
5												22	0	0	0
6												φ	1	4	4
7				7 14 1	2 4 4	11 22 7	2 4 2	3 6 2	6 12 3	3 6 1		33	2	66	132
8												φ	3	12	36
9				4 8 1		4 8 3	8 16 5	12 24 4	16 32 1	20 40 2		20	4	80	320
10							10 20 1	15 30 2	20 40 2			5	5	25	125
f	φ	6	8	14	18	25	21	21	12	4	2	131		Σ = 7	Σ = 1.169
d		-4	-3	-2	-1	0	1	2	3	4	5	7		Σ ay = 508	
fd		-24	-24	-28	-18	φ	21	42	36	16	10	31		+ 550	
fd <sup>2</sup>		96	72	56	18		21	84	108	64	50	509		- 42	
														508	

3: part (10)

$G_x = 2,98$   
 $G_y = 4 \times 2,07$   
 $N_x = 5,1$   
 $N_y = 20,23$

$G_x = \sqrt{\frac{1.169}{131} - \left(\frac{7}{131}\right)^2}$   
 $G_x = \sqrt{8,32 - 0,0025}$   
 $G_x = \sqrt{8,9175}$   
 $G_x = 2,98$

$n = \frac{508}{131} - 0,23 \times 0,05$   
 $n = \frac{2,98 \times 2,07}{2,98 \times 2,07}$

$n = \frac{3,87 - 0,0115}{0,1686}$

$n = \frac{3,8585}{0,1686}$

$n = 0,62$

$G_2 = \frac{1 - 0,3844}{11,44}$

$G_2 = \frac{0,6156}{11,44}$

$G_2 = 0,05$

$3G_2 = 0,15$

$0,47 \text{ --- } 0,77$



Total (y)

	0-3	4-7	8-11	12-15	16-19	20-23	24	25	26-29	30-33	34-37	38-41	f	d	fd	fd <sup>2</sup>
0		6 <sup>1</sup> 4	12 <sup>2</sup> 2	3 <sup>1</sup> 1	4 <sup>1</sup> 1								9	-4	-36	144
1		12 <sup>1</sup> 1	7 <sup>1</sup> 1	13 <sup>2</sup> 2	12 <sup>2</sup> 2								8	-3	-24	72
2		3 <sup>1</sup> 1	6 <sup>1</sup> 1	24 <sup>2</sup> 2	14 <sup>2</sup> 2								23	-2	-46	92
3													8	-1	0	0
4													15	0	0	0
5				4 <sup>1</sup> 1	21 <sup>2</sup> 2		6 <sup>1</sup> 1	4 <sup>1</sup> 1	6 <sup>1</sup> 1				15	1	15	15
6			6 <sup>1</sup> 1	4 <sup>1</sup> 1	6 <sup>1</sup> 1		6 <sup>1</sup> 1	23 <sup>2</sup> 2	6 <sup>1</sup> 1	8 <sup>1</sup> 1			19	2	38	76
7				3 <sup>1</sup> 1	13 <sup>2</sup> 2		3 <sup>1</sup> 1	30 <sup>2</sup> 2	27 <sup>2</sup> 2	12 <sup>1</sup> 1			20	3	60	180
8													8	4	0	0
9				5 <sup>1</sup> 1	10 <sup>2</sup> 2		5 <sup>1</sup> 1	15 <sup>2</sup> 2	40 <sup>2</sup> 2	50 <sup>2</sup> 2			16	5	80	400
10				6 <sup>1</sup> 1	16 <sup>2</sup> 2			12 <sup>1</sup> 1	24 <sup>2</sup> 2	36 <sup>2</sup> 2			6	6	36	216
f	9	6	8	14	18	25	21	21	12	4	2		131			$\sum u = 123$ $\sum u^2 = 195$
d	-5	-4	-3	-2	-1	0	1	2	3	4	5					$\sum ux = 525$
fd	-0	-24	-24	-28	-18	0	21	42	36	16	10		$\sum = 31$			$+576$ $-51$ $525$
fd <sup>2</sup>	0	96	72	56	18	0	21	84	108	64	50		$\sum = 569$			

$$\sigma = i \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$$

$$\sigma = \sqrt{\frac{1195}{131} - \left(\frac{123}{131}\right)^2}$$

$$\sigma = \sqrt{9,12 - 0,8649}$$

$$\sigma = \sqrt{8,2551}$$

$$\sigma = 2,87$$

$$\sigma = 2,87$$

$$\sigma = i \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$$

$$\sigma = 4 \sqrt{\frac{569}{131} - \left(\frac{31}{131}\right)^2}$$

$$\sigma = 4 \sqrt{4,34 - 0,0527}$$

$$\sigma = 4 \sqrt{4,2871}$$

$$\sigma = 4 \times 2,071$$

$$\sigma = 8,28$$

$M_u = 4,94$   
 $M_y = 22,23$

$$r = \frac{\sum u_y - C_u C_y}{\sigma_u \sigma_y}$$

$$r = \frac{525 - 0,93 \times 0,23}{2,87 \times 2,07}$$

$$r = \frac{4,007 - 0,2139}{5,9409}$$

$$r = \frac{3,7831}{5,9409}$$

$$r = 0,63$$

$$\sigma_r = \frac{1-r^2}{\sqrt{r}}$$

$$\sigma_r = \frac{1-0,3969}{11,44}$$

$$\sigma_r = \frac{0,6031}{11,44}$$

0,48 ————— 0,78

$$\sigma_r = 0,05$$

$$3 \times \sigma_r = 0,05 \times 3 = 0,15$$

α.



total

	0	1	2	3	4	5	6	7	8	9	10	f	d	fd	fd <sup>2</sup>	
0		12	9	6	3								59	-3	-177	531
1		8	6	4	2								16	-2	-32	64
2			2	2	1								5	-1	-5	5
3				1	2								4		-214	222
4					1								5	1	5	5
5						2							2	2	4	8
6							3						5	3	15	45
7								4					7	4	28	112
8									5				5	5	25	125
9										6			16	6	96	576
10											7		7	7	49	343
f	-	6	8	14	18	25	21	21	12	4	2	131			1814	
d		-4	-3	-2	-1		1	2	3	4	5					Σxy = 534
fd		-24	-24	-28	-18	-94	21	42	36	16	10	31				
fd <sup>2</sup>		96	72	56	18		21	84	108	64	50	569				

$$\sigma_x = \frac{\sqrt{1814 \times 131 - 8^2}}{131} = \frac{\sqrt{237.570}}{131} = \frac{487.41}{131} = 3.72$$

$$\sigma_y = 4 \times \frac{\sqrt{569 \times 131 - 31^2}}{131} = 4 \times \frac{\sqrt{440.39 - 761}}{131} = 4 \times \frac{271.25}{131} = 4 \times 2.07$$

Σxy

$$r = \frac{\Sigma xy}{N \cdot \sigma_x \cdot \sigma_y} = \dots$$

$$r = \frac{534}{131} \cdot \frac{8 \times 31}{131 \times 131} = \frac{534 \times 131 - 248}{17.161} = \frac{7.7004}{7.7004} = 1$$

$$r = \frac{69954 - 248}{17161} = \frac{4,0618}{7,7004} = 0.53$$

$$\sigma_r = \frac{1 - 0.2809}{\sqrt{N}} = \frac{0,7191}{11,44} = 0,06$$

$$3\sigma_r = 0.18$$

$$0.35 \text{ --- } 0.71$$



4

Auto-correlação.

	3-3	4-7	8-11	12-15	16-17	20-23	24-27	28-31	32-35	36-37	48-49	j	k	jk	jk <sup>2</sup>	
3-3																
4-7	30												4	-4	-16	64
8-11													8	-3	-6	12
12-15													12	-2	-14	22
16-17													17	-1	-7	7
20-23													12			
24-27													15	1	15	15
28-31													9	2	12	36
32-35													7	3	21	63
36-37													1	4	4	16
48-49													1	5	5	25
j	2	6	7	11	11	1	11	5	3	1	65				$\sum = 20$	$\sum = 72$
k	-4	-3	-2	-1		1	2	3	4	6					$\sum = -14$	
jk	-8	-12	-14	-11		8	22	15	12	5					$\sum = 11$	$\frac{-106}{+72}$ $-14$
jk <sup>2</sup>	32	54	28	11		8	44	45	48	25					$\sum = 275$	

$$\sigma_x = 3 \sqrt{\frac{276}{65} - \left(\frac{20}{65}\right)^2}$$

$$\sigma_x = 3 \sqrt{4,18 - 0,97}$$

$$\sigma_x = 3 \sqrt{4,01}$$

$$\sigma_x = 3 \times 2,02$$

$$\sigma_y = 3 \sqrt{\frac{275}{65} - \left(\frac{11}{65}\right)^2}$$

$$\sigma_y = 3 \sqrt{4,54 - 0,0289}$$

$$\sigma_y = 3 \sqrt{4,511}$$

$$\sigma_y = 3 \times 2,12$$

$$r = \frac{\frac{-17}{65} - 0,3 \times 0,17}{2,02 \times 2,12}$$

$$r = \frac{-0,26 - 0,051}{4,2824}$$

$$r = \frac{0,311}{4,2824}$$

$$r = -0,07$$

$$\sigma_x = \frac{1 - (-0,07)^2}{\sqrt{0,5}}$$

$$\sigma_x = \frac{0,7951}{2}$$

$$\sigma_x = 0,12$$

$$3\sigma_x = 0,36$$

$$-0,43 \leftarrow \rightarrow 0,27$$



# Auto-Correlação

4

	0-3	4-7	8-11	12-15	16-19	20-23	24-27	28-31	32-35	36-39	40-43	f	d	fd	fd <sup>2</sup>
0-3												1	0	0	0
4-7								14				2	-4	-8	32
8-11		12					9					6	3	-18	54
12-15				2			4					7	-2	-14	28
16-19				4	4		1	2				10	-1	-10	10
20-23															
24-27		3						2	3			8	1	8	8
28-31				4	2		7	4			10	12	2	24	48
32-35				6	6		6					5	3	15	45
36-39							4					3	4	12	48
40-43					10							1	5	5	25
f	0	4	2	7	7	14	13	9	7	1	1	65		$\sum = 14$	$\sum = 298$
fd		-4	-3	-2	-1		1	2	3	4	5			$\sum = 18$	
fd <sup>2</sup>		16	9	14	7		13	18	21	4	5			$\sum = 93$	
fd <sup>3</sup>		64	18	28	7		13	36	63	16	25			$\sum = 70$	

$$\sigma_u = i \sqrt{\frac{298}{65} - \left(\frac{14}{65}\right)^2}$$

$$\sigma_u = 3 \sqrt{4,6 - 0,441}$$

$$\sigma_u = 3 \sqrt{4,159}$$

$$\sigma_u = 3 \times 2,04$$

$$\sigma_u = 6,33$$

$$\sigma_y = i \sqrt{\frac{270}{65} - \left(\frac{14}{65}\right)^2}$$

$$\sigma_y = 3 \sqrt{4,15 - 0,0727}$$

$$\sigma_y = 3 \sqrt{4,0771}$$

$$\sigma_y = 3 \times 2,01$$

$$\sigma_y = 6,03$$

$M_u = 270$   
 $M_y = 270,4$

$$r = \frac{-17}{65} - 0,21 \times 0,27$$

$$r = \frac{2,13 \times 2,01}{2,13 \times 2,01}$$

$$r = \frac{-0,26 - 0,0567}{4,2813}$$

$$r = \frac{-0,3167}{4,2813}$$

$$r = -0,07$$

$$\sigma_z = \frac{1 + 0,07^2}{\sqrt{65}}$$

$$\sigma_z = \frac{1 - 0,0049}{1,8}$$

$$\sigma_z = \frac{0,9951}{1,8}$$

$$\sigma_z = 0,55$$

$$3\sigma_z = 1,65$$

$$r \pm 3\sigma_z$$

$$-0,07 \pm 1,65$$

12

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